Math 1B Quiz 1 Version 1
Tue Jan 12, 2016

NAME YOU ASKED TO BE CALLED IN CLASS:

SCORE: _____ / 30 POINTS

1. NO CALCULATORS OR NOTES ALLOWED
2. SHOW PROPER CALCULUS-LEVEL WORK
3. SIMPLIFY ALL ANSWERS

For this question, you may use the formulae for $\frac{d}{dx}\sinh x$, $\frac{d}{dx}\cosh x$ and/or $\frac{d}{dx}\tanh x$ without proving them. **SCORE: _____ / 8 PTS**

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the exponential formula for sech x**, prove the formula for $\frac{d}{dx}\operatorname{sech} x$.

$$\frac{d}{dx}\operatorname{sech}x = \frac{d}{dx}\left(\frac{1}{\cosh x}\right) = \frac{\cosh x \cdot \frac{d(1)}{dx} - 1 \times \frac{d(\cosh x)}{dx}}{(\cosh x)^2}$$

$$= \frac{0 - \sinh x}{\cosh^2 x} = -\frac{\sinh x}{\cosh x} \cdot \frac{1}{\cosh x} = -\tanh x \cdot \operatorname{sech} x$$

(*) (1)

[b] **Without using the logarithmic formula for $\tanh^{-1} x$**, prove the formula for $\frac{d}{dx}\tanh^{-1} x$.

$\frac{d}{dx}\tanh^{-1}x$. let $y = \tanh^{-1}x \rightarrow x = \tanh y$. (1)

$$\frac{d}{dx}(x) = \frac{d(\tanh y)}{dx} = \operatorname{sech}^2 y \cdot \frac{dy}{dx}$$

$\frac{d}{dx}(x) = 1$ (1 1/2) $\operatorname{sech}^2 y \cdot \frac{dy}{dx} = 1$, so $\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y}$

since $1 - \tanh^2 y = \operatorname{sech}^2 y$. (1) $\rightarrow$ so $\frac{dy}{dx} = \frac{1}{1-\tanh^2 y} = \frac{1}{1-x^2}$

so. $\frac{d}{dx}\tanh^{-1}x = \frac{dy}{dx} = \frac{1}{1-x^2}$ (1 1/2)

Find $\lim_{x\to\infty} \coth x$ **algebraically**. **SCORE: 3 / 3 PTS**

$$\lim_{x\to\infty} \coth x = \lim_{x\to\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} = \lim_{x\to\infty} \frac{e^{2x}+1}{e^{2x}-1}$$ (1)

$x \to \infty \rightarrow e^{2x} \to \infty$. $e^{2x}+1 \to \infty$, $e^{2x}-1 \to \infty$

$\therefore \lim_{x\to\infty} \frac{e^{2x}+1}{e^{2x}-1} \left(\frac{\infty}{\infty}\right)$

(1) $= \lim_{x\to\infty} \frac{2e^{2x}}{2e^{2x}} = 1$ (1)

If $\coth x = -5$, find $\sinh x$.

SCORE: _____ / 4 PTS

$\cosh^2 x - \sinh^2 x = 1$ (1/2)

$1 - \tanh^2 x = \operatorname{sech}^2 x = \frac{1}{\cosh^2 x}$

$1 - \frac{1}{\coth^2 x} = \operatorname{sech}^2 x = \frac{1}{\cosh^2 x}$

$1 - \frac{1}{(-5)^2} = \frac{1}{\cosh^2 x}$ (1/2)

$1 - \frac{1}{25} = \frac{1}{\cosh^2 x} = \frac{24}{25}$ (1/2)

$\cosh^2 x = \frac{25}{24}$ $\cosh x = \pm\sqrt{\frac{25}{24}} = \pm\frac{5}{2\sqrt{6}} = \pm\frac{5\sqrt{6}}{12}$ (*)

because $\cosh x > 0$ for all x.

so $\cosh x = \frac{5\sqrt{6}}{12}$ (1/2)

so $\frac{\cosh x}{\sinh x} = \coth x$

$\sinh x = \frac{\cosh x}{\coth x} = \frac{\frac{5\sqrt{6}}{12}}{-5} = -\frac{\sqrt{6}}{12}$ (*) (1/2)

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook.

NOTE: This is NOT a question about derivatives.

SCORE: _____ / 5 PTS

let $y = \sinh^{-1} x$ $\quad x = \sinh y$.

(1) $x = \frac{e^y - e^{-y}}{2} = \frac{(e^y - e^{-y}) \times e^y}{2 \times e^y} = \frac{e^{2y} - 1}{2e^y}$

(*) $2xe^y = e^{2y} - 1$

$e^{2y} - 2xe^y - 1 = 0$. let $z = e^y$

$e^{2y} - 2xe^y - 1 = 0 \rightarrow$ so $z^2 - 2xz - 1 = 0$ (1/2)

$z = \frac{2x \pm \sqrt{4x^2+4}}{2}$ (1) $= x \pm \sqrt{x^2+1}$ (1/2)

because $z = e^y > 0$ (1/2)

$\therefore z = x + \sqrt{x^2+1}$ (1/2)

so $e^y = x + \sqrt{x^2+1}$

$\rightarrow y = \ln(x + \sqrt{x^2+1})$

$\rightarrow$ so $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ (1/2)

Find $\frac{d}{dx} x^2 \cosh^{-1}(x^5)$. Simplify your final answer as a single fraction.

SCORE: 4 / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$\frac{d}{dx} x^2 \cosh^{-1}(x^5) = \cosh^{-1}(x^5) \times \frac{d(x^2)}{dx} + x^2 \cdot \frac{d(\cosh^{-1}(x^5))}{dx}$

$= \cosh^{-1}(x^5) \cdot 2x + x^2 \cdot \frac{1}{\sqrt{x^{10}-1}} \times 5x^4$

$= 2x\cosh^{-1}(x^5) + \frac{5x^6}{\sqrt{x^{10}-1}}$ (1) (1) (2)

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[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS: [1] f [2] c [3] b [4] d [5] d [6] e

[1] If you score 110 points on Midterm 1, 130 points on Midterm 2 and 135 points on Midterm 3, which midterm score(s) will be changed, and to what value ?
(HINT: You are encouraged to start studying regularly early in the quarter.)

[a] Midterm 1's score will be changed to 135 (the highest midterm score)
[b] Midterm 1's score will be changed to $(110 + 130 + 135) \div 3 = 125$ (the average of all midterm scores)
[c] Midterm 1's score will be changed to $(110 + 130) \div 2 = 120$ (the average of Midterm 1's and Midterm 2's scores)
[d] Midterm 1's score will be changed to $(110 + 135) \div 2 = 122.5$ (the average of Midterm 1's and the highest midterm's scores)
[e] Midterm 1's score will be changed to $(110 + 135) \div 2 = 122.5$ and Midterm 2's score will be changed to $(130 + 135) \div 2 = 132.5$ (the average of each midterm's and the highest midterm's score)
[f] no midterm scores will be changed

[2] If you score 145 points on Midterm 1, 125 points on Midterm 2 and 150 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

[a] Midterm 2's score will be changed to 150 (the highest midterm score)
[b] Midterm 2's score will be changed to $(125 + 145 + 150) \div 3 = 140$ (the average of all midterm scores)
[c] Midterm 2's score will be changed to $(125 + 145) \div 2 = 135$ (the average of Midterm 2's and Midterm 1's scores)
[d] Midterm 2's score will be changed to $(125 + 150) \div 2 = 137.5$ (the average of Midterm 2's and the highest midterm's scores)
[e] Midterm 1's score will be changed to $(145 + 150) \div 2 = 147.5$ and Midterm 2's score will be changed to $(125 + 150) \div 2 = 137.5$ (the average of each midterm's and the highest midterm's score)
[f] no midterm scores will be changed

[3] Which statement below regarding attendance is **false** ?

[a] Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.
[b] Arriving late on a quiz or midterm day will not be counted as late.
[c] Unexcused early departures are considered absences.
[d] If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
[e] Attendance policies will not apply to you if you score more than 80% on every midterm.

[4] How much of your learning does the instructor believe comes from your daily reading and homework ?

[a] reading = 10%, homework = 30% ➡ combined = 40%
[b] reading = 15%, homework = 35% ➡ combined = 50%
[c] reading = 15%, homework = 45% ➡ combined = 60%
[d] reading = 20%, homework = 50% ➡ combined = 70%
[e] reading = 25%, homework = 55% ➡ combined = 80%

[5] Proper use of the textbook for this class includes

[a] understanding all the terminology used in the book
[b] working out the given examples yourself and checking that you are able to get the same results as the book
[c] reading the sections of the textbook before the corresponding lecture
[d] all of the previous answers [a], [b] and [c]
[e] some, but not all, of the previous answers [a], [b] and [c]

[6] Which statement below regarding tests (quizzes, midterms, final exam) is **false** ?

[a] If you continue writing on your test after the stated ending time, you will receive a 0 for that test.
[b] There are no make-ups for missed quizzes.
[c] The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.
[d] If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.
[e] If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.